MATH 140A Review: Sequences and Series

Facts to Know:

How do we add an infinite list of numbers?

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \cdots$$

Answer: Take a look at the partial sums:

$$\begin{aligned}
S_1 &= a_1 \\
S_2 &= a_1 + a_2 \\
S_3 &= a_1 + a_2 + a_3 \\
&\vdots \\
S_N &= a_1 + a_2 + a_3 + \cdots + a_N = \sum_{n=1}^{N} a_n
\end{aligned}$$

If the limit of the sequence $\{S_N\}_{N=1}^{\infty}$ exists, then define

$$\sum_{n=1}^{\infty} a_n = \lim_{N\to\infty} S_N = \lim_{N\to\infty} \sum_{n=1}^{N} a_n$$

and say that the series converges. Otherwise, the series diverges.

Example: Determine if the series

$$\sum_{n=9}^{\infty} \ln \frac{1+\frac{1}{n}}{1+\frac{1}{n-1}}$$

converges? If it converges, what does the series add up to?

Facts to Know:

Let $r \in \mathbb{R}$. The sequence $a_n = r^n$ is called the *geometric sequence*.

$$\lim_{n\to\infty} r^n = \begin{cases} 0 & |r| < 1, \\ 1 & r = 1, \\ \text{DNE} & r \leq -1, \\ \infty & 1 < r \end{cases}.$$

The *geometric series*

$$\sum_{n=0}^{\infty} r^n = \frac{1}{1-r},$$

converges for $|r| < 1$ and diverges otherwise.

Example: Determine if the following series converges. If so, what is the sum?

$$\sum_{n=2}^{\infty} 3 \cdot \frac{1}{4^n}.$$